Map Coloring Problem

You are the publisher of a new edition of the world atlas. As you prepare the different maps for printing, you need to make sure that countries adjacent to each other (sharing a common border) are given different colors.

1. For the following two maps, decide how to color each of the five countries (regions) so that no two adjacent countries are colored the same. Treat the outside region as a single country (perhaps it represents an ocean colored blue). Assume that every country is composed of a single contiguous region (for example, you treat Alaska and Hawaii as separate regions when constructing a map of the world).

2. How many colors did you use to color each map?

3. REFLECTION: Did you use fewer colors than anyone else? If not, describe how you can adjust your map to use fewer colors. Yes, how are you confident that the fewest colors have been used that can be?

4. If you want to color each map using the least number of colors (still keeping adjacent regions separate colors), how many colors are needed for each map?

5. Create a map that requires the use of three colors.
6. Create a map with at least four different regions that could be colored with two colors.

7. **EXTENSION:** Create a map that needs five colors. What is the largest number of colors required to color any map, that keeps adjacent regions separate? Justify your response.

As long as each country is connected, four will always work.
Networks and Graphs: Graph Coloring
VII.C Student Activity Sheet 10: Coloring Maps and Scheduling

Creating Graphs from Maps

1. Revisit the map coloring exercises from Student Activity Sheet 9 in terms of graphs. For example, Map I can be represented by the following graph. The graph should include a vertex for each country (or region) in your map. If two countries share a border and need to be colored differently, the graph shows an edge between the vertices that represent them.

After studying the relationship between Map I and the graph for Map I, create a graph that represents Map II.
Networks and Graphs: Graph Coloring
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2. Restate the Map Coloring problem from Student Activity Sheet 9 in terms of a Graph Coloring problem.

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3. Create a graph that requires three colors.

4. Create a graph that could be colored with two colors.

5. What types of graphs can always be colored with two colors?

6. **EXTENSION:** Create a graph that needs five colors, and then draw the associated map.

7. **REFLECTION:** When might a graph not correspond to a map?

8. The **chromatic number** of a graph is the minimum number of colors needed to color each vertex in such a way that any two vertices sharing an edge are a different color. Provide examples of graphs that have chromatic numbers of 3 and 4.

9. Give an example of a graph with 20 vertices that has a chromatic number of 2. Does your graph have any cycles? (Recall: A cycle is a path through the graph that starts and ends at the same vertex and does not reuse any edges.)