Creating Graphs from Maps

1. Revisit the map coloring exercises from Student Activity Sheet 9 in terms of graphs. For example, Map I can be represented by the following graph. The graph should include a vertex for each country (or region) in your map. If two countries share a border and need to be colored differently, the graph shows an edge between the vertices that represent them.

After studying the relationship between Map I and the graph for Map I, create a graph that represents Map II.
2. Restate the Map Coloring problem from Student Activity Sheet 9 in terms of a Graph Coloring problem.

   You are the publisher of a new edition of the world atlas. As you prepare the different maps for printing, you need to make sure that countries adjacent to each other (sharing a common border) are given different colors.

3. Create a graph that requires three colors.

4. Create a graph that could be colored with two colors.

5. What types of graphs can always be colored with two colors?

6. EXTENSION: Create a graph that needs five colors, and then draw the associated map.

7. REFLECTION: When might a graph not correspond to a map?

8. The chromatic number of a graph is the minimum number of colors needed to color each vertex in such a way that any two vertices sharing an edge are a different color. Provide examples of graphs that have chromatic numbers of 3 and 4.

9. Give an example of a graph with 20 vertices that has a chromatic number of 2. Does your graph have any cycles? (Recall: A cycle is a path through the graph that starts and ends at the same vertex and does not reuse any edges.)
Networks and Graphs: Graph Coloring
VII.C Student Activity Sheet 10: Coloring Maps and Scheduling

Scheduling Problem

Mrs. Jacobs, the new principal at Riverdale High School, wants to make a good impression by offering a lot of new exciting classes for her students. The principal plans to use her knowledge of graph theory to determine when each class will be offered.

Since she is trying to make her students happy, Mrs. Jacobs does not want to offer two different classes at the same time if there are students wanting to take both. She decides to construct a graph in the following way: Each class is represented by a vertex and if there is a student interested in two classes, those two vertices are connected by an edge.

10. Suppose there are five classes (A, B, C, D, and E) and only five students wishing to take the following classes:
   - Jason wants to take Classes A and E.
   - Emory wants to take Classes B, C, and E.
   - Felicity wants to take Classes A and D.
   - Geoff wants to take Classes B and C.
   - Hilary wants to take Classes D and E.
   
   Construct the graph for the principal.

11. Find the chromatic number of the graph, and color the graph using the least number of colors.

12. How can the graph coloring solution help the principal with her scheduling problem?
13. **EXTENSION:** Select another situation that might be modeled with colored graphs. Several suggestions are described to stimulate your research. Prepare a short presentation of your findings to share with the class.

The notion of coloring graphs can be used to solve a variety of problems involving various types of conflicts over space or time. Some examples include the following:

- **Conflict over time:** Virtually any type of scheduling problem such as appointments or job duties based on an individual’s qualifications.

- **Conflict over space:** 1) Create several terrariums to display a variety of plants and reptiles. Certain reptiles may not get along with others, and certain plants should not be placed in terrariums with certain reptiles. Based on a set of compatibility conditions, you could decide the minimum number of terrariums necessary for the exhibit. 2) Radio stations that are within a certain distance of each other cannot be assigned the same broadcasting frequency. Given several radio stations and the distances between each pair, determine the minimum number of distinct frequencies necessary to allow all stations to operate.

- **Other conflicts:** Put a roomful of people into small working groups. Each individual may have a list of others with whom he/she does not work well, thereby disallowing them to share a group. Given each person’s “Cannot Work With” list, how many groups are necessary?

- **Chemistry:** Certain chemicals cannot be stored with other chemicals. For example, to answer the question regarding how many storage facilities are required to house the following chemicals, graph coloring can be helpful.

<table>
<thead>
<tr>
<th>Chemicals</th>
<th>Cannot be stored with</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2, 5, 7</td>
</tr>
<tr>
<td>2</td>
<td>1, 3, 5</td>
</tr>
<tr>
<td>3</td>
<td>2, 4</td>
</tr>
<tr>
<td>4</td>
<td>3, 7</td>
</tr>
<tr>
<td>5</td>
<td>1, 2, 6, 7</td>
</tr>
<tr>
<td>6</td>
<td>5</td>
</tr>
<tr>
<td>7</td>
<td>1, 4, 5</td>
</tr>
</tbody>
</table>
4. What is the minimum amount of time required to perform all five activities?

5. Which path corresponds to this minimum time? Which activities are along this path?

6. Which activities could take a little longer to complete without affecting the total completion time?

Scheduling classes in college can be very similar to the previous scenario. Over four years, there are certain classes that you must take, and many classes have prerequisites—classes that must be taken first. Suppose you need to take the following classes with the identified prerequisites.

<table>
<thead>
<tr>
<th>Class</th>
<th>Prerequisite</th>
</tr>
</thead>
<tbody>
<tr>
<td>Calculus I</td>
<td>None</td>
</tr>
<tr>
<td>Calculus II</td>
<td>Calculus I</td>
</tr>
<tr>
<td>Physics I</td>
<td>Calculus I</td>
</tr>
<tr>
<td>Physics II</td>
<td>Physics I</td>
</tr>
<tr>
<td>Psychology I</td>
<td>None</td>
</tr>
<tr>
<td>Speech</td>
<td>None</td>
</tr>
<tr>
<td>Argument and Debate</td>
<td>Speech</td>
</tr>
</tbody>
</table>

7. Construct an activity graph for this situation using the following rules:
   - Create Start and Finish squares.
   - Any activity that can be performed right away is connected to Start.
   - Activity A is connected to Activity B by an arrow only when Activity A needs to be performed directly before Activity B.
   - Any activity that does not precede any other activity can be connected to Finish.

8. Identify the longest path from Start to Finish. How long is this path?

9. If each class is a semester long, how many semesters are needed to take all these classes?
Networks and Graphs: Program Evaluation and Review Technique (PERT) Charts
VII.D Student Activity Sheet 11: Activity Graphs

10. If you want to finish these classes as soon as possible, which classes should you not delay taking?

11. How long could you wait to take Psychology I without delaying your overall program of classes?

12. How long could you wait to take Speech without delaying your overall program of classes?

13. Given any activity graph like the previous ones, explain how you would determine the minimum time required to perform all activities.

14. Activities that cannot be delayed without increasing the minimum time for completion are called critical activities. Given any activity graph like the previous ones, explain how you would determine which activities are critical activities.

For the following activity graph, the times given for each activity are in hours.

15. Determine the minimum time required to complete all the activities shown in the graph.

16. Which activities are critical activities?

17. How long could Activity F be delayed without affecting the overall completion time?

18. How long could Activity D be delayed without affecting the overall completion time?
19. What if Activities F and D were both delayed?

20. REFLECTION: Can you find a formula that determines how long an individual activity could be delayed without affecting the total completion time for all the activities?

Sometimes the time to complete an activity is given by two numbers: the estimate for a minimum completion time and the estimate for a maximum completion time. How would having two possible completion times affect your analysis?

21. EXTENSION: Design a chart to represent the planning and design of a particular event at your school, (for example, Project Graduation, prom, fundraiser, community project). Prepare a short presentation including appropriate visuals to share with the class.
 Networks and Graphs: Program Evaluation and Review Technique (PERT) Charts
VII.D Student Activity Sheet 12: Building a Robot

You are leading a group that is designing and building a robot; the group is divided into several teams. The following table indicates the different activities that go into this complex process, which teams are in charge of which activities, the number of individuals from that team dedicated to that activity, how long the activity will likely take, and which activities must be completed before an activity can be started.

<table>
<thead>
<tr>
<th>Activity</th>
<th>Time to Complete (in Weeks)</th>
<th>Must First Finish Activity...</th>
<th>Team</th>
<th>No. of Members</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sensory Program (SP)</td>
<td>4</td>
<td>None</td>
<td>Computer programmers</td>
<td>3</td>
</tr>
<tr>
<td>Artificial Intelligence (AI)</td>
<td>5</td>
<td>None</td>
<td>Computer programmers</td>
<td>3</td>
</tr>
<tr>
<td>Motion (M)</td>
<td>10</td>
<td>SP, AI</td>
<td>Computer programmers</td>
<td>3</td>
</tr>
<tr>
<td>Voice System (VS)</td>
<td>3</td>
<td>AI</td>
<td>Computer programmers</td>
<td>3</td>
</tr>
<tr>
<td>Eye Design (ED)</td>
<td>5</td>
<td>SP</td>
<td>Engineers</td>
<td>3</td>
</tr>
<tr>
<td>Arm and Leg Design (ALD)</td>
<td>3</td>
<td>M</td>
<td>Engineers</td>
<td>3</td>
</tr>
<tr>
<td>Body Design (BD)</td>
<td>4</td>
<td>None</td>
<td>Engineers</td>
<td>3</td>
</tr>
<tr>
<td>Head Assembly (HA)</td>
<td>5</td>
<td>VS, ED</td>
<td>Technicians</td>
<td>3</td>
</tr>
<tr>
<td>Torso Assembly (TA)</td>
<td>2</td>
<td>BD</td>
<td>Technicians</td>
<td>3</td>
</tr>
<tr>
<td>Arm and Leg Assembly (ALA)</td>
<td>8</td>
<td>ALD</td>
<td>Technicians</td>
<td>3</td>
</tr>
<tr>
<td>Appearance (A)</td>
<td>2</td>
<td>TA, ALA</td>
<td>Technicians</td>
<td>3</td>
</tr>
<tr>
<td>Final Assembly (FA)</td>
<td>4</td>
<td>HA, TA, ALA</td>
<td>Technicians</td>
<td>3</td>
</tr>
</tbody>
</table>
Networks and Graphs: Program Evaluation and Review Technique (PERT)
Charts
VII.D Student Activity Sheet 12: Building a Robot

1. Using the information in the first three columns of the table, build an activity graph. Include Start and Finish boxes.

2. Assuming the times given for each activity are accurate, what is the minimum time required to design and build the robot?

3. At what point in the timeline does each activity (for the completion of the entire robot) begin and end?

4. Which of the 12 activities are critical activities?

5. **EXTENSION:** Since any delay in the completion time for critical activities results in a longer total completion time, these activities may need extra people assigned to them. Suppose you can reassign team members to an activity according to the following guidelines:
   - No one can work on an activity outside of his/her team. For example, a computer programmer must be assigned to Activity 1, 2, 3, or 4 and cannot be assigned to any of the other activities.
   - Every activity must have at least one person assigned to it at all times.
   - An activity that receives extra help can be completed 1 week earlier for each additional person assigned to it.
   - An activity cannot be completed in less than 1 week, even if more people are assigned to it.
   - An activity takes 1 week longer to complete for each person removed from the original group.

   a. If you could reassign one person, how would you do it? How does the reassignment affect the total completion time?

   b. If you could reassign two people, how would you do it? How do the reassignments affect the total completion time?

   c. If you could reassign any number of people, how would you do it? How do the reassignments affect the total completion time?

6. **REFLECTION:** Could the total completion time be further improved by allowing people to work on activities outside of their official team designation? Justify your response with appropriate reasoning.